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COMPRESSION BY LINERS OF PLASMA-FIELD CONFIGURATIONS WITH BETA --ETC(U)

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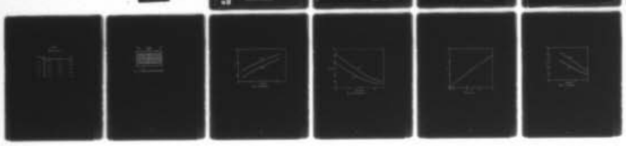

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Compression by Liners of Plasma-Field Configurations with $\beta < 1$

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April 1977

NAVAL RESEARCH LABORATORY
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This report derives the thermonuclear yield from a cylindrical plasma with $\beta < 1$ (specifically, a reversed-field theta-pinch configuration) being compressed by an imploding liner. Since magnetic field is mixed with plasma, the average plasma density, and hence the fusion output are reduced in comparison with the $\beta = 1$ systems considered previously. The trajectory of the liner is also modified due to the increased γ (ratio of specific heats) of the plasma-field mixture. For the same peak field and system size, the value of Q (ratio of fusion output to system energy) is reduced to a fraction

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beta gamma

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20. Abstract (Continued) beta

s of its value in the $\beta = 1$ case, where s depends upon the initial pressure distribution in the plasma, and upon the compression ratio; over the likely range of parameters, $0.1 < s < 0.3$. The significance of this result for the design of LINUS fusion systems is discussed.

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COMPRESSION BY LINERS OF PLASMA-FIELD CONFIGURATIONS WITH $\beta < 1$

I. Introduction

A simple dynamic model of a cylindrical, incompressible liner imploding into a $\beta = 1$ plasma was developed early in the LINUS program[1] in order to derive the basic scaling laws for power-producing systems. This model has recently been further developed to cover compressible liners,[2] but the assumption of a $\beta = 1$ plasma was retained. Since the thick, captive liquid liners which now form the basis of the LINUS concept[3] are inevitably rather slow, axial confinement of the plasma is essential, and one method of achieving this is by means of a closed field-line system such as the reversed-field theta-pinch (or belt-pinch, if an azimuthal field is included). In this system magnetic field and plasma are mixed, that is to say, $\beta < 1$; the most important consequence is that, for a given Q, the radius of the system will be greater than in the $\beta = 1$ case. Clearly it is desirable to choose conditions that minimize this increase, in order to maintain our claim that LINUS reactors will be very compact devices.

The principal relationship obtained from the incompressible model of Ref. 1 was

$$Q = B_o r_o \rho^{\frac{1}{2}} P(b, T_o) \qquad (1)$$

Note: Manuscript submitted April 5, 1977.

where Q = nuclear energy released in one cycle/plasma energy at peak compression;

B_o = buffer magnetic field between liner and plasma at peak compression (assumed of negligible extent);

r_o = minimum liner radius;

ρ = density of liner material;

T_o = plasma temperature at peak compression;

b = a dimensionless thickness parameter, defined by $A = \pi b^2 r_o^2$, where A is the cross-sectional area of the liner.

The function $P(b, T_o)$ was evaluated by integrating the thermonuclear reaction rate over a compression-expansion cycle; the plasma was assumed to have uniform density and temperature.

When plasma and field are mixed in a $\beta < 1$ system, the basic relationship (1) still holds, but the value of P is modified as the result of a number of effects, namely:

1) The average plasma density is reduced, reducing the fusion reaction rate and hence decreasing P.

2) The system energy, for a given pressure, is reduced since now $5/3 < \gamma < 2$, compared to $\gamma = 5/3$ for the $\beta = 1$ case. This increases P.

3) A consequence of (2) is that the expansion velocity is reduced, and the dwell time increased. This further increases P.

4) The increase in γ causes the pressure to fall more rapidly as the system expands. This decreases P.

Of these effects, (1) is dominant and results in a net decrease in P. However the other effects are all significant and will be included in the following discussion.

II. Compression of a Reversed-Field Pinch

The reversed-field theta-pinch (Fig. 1) is a long cylindrical configuration of radius R in which the magnetic field B is in the z-direction over most of the length, and goes from $-B_1$ on axis to $+B_1$ at $r = R$. The field lines close at the ends, and for the purpose of this report we assume that this provides perfect axial confinement of plasma. The radial distribution of plasma pressure $p = 2nkT$ is determined by the pressure balance equation

$$p + B^2/8\pi = p_1 = B_1^2/8\pi \tag{2}$$

There is a cylindrical neutral surface in the center of the plasma at which $B = 0$ and here the local β, defined by $\beta = 2nkT/p_1$ will be unity. Elsewhere $\beta < 1$, falling to zero at $r = 0$ and $r = R$.

Consider the system initially at pressure p_i and take an element of plasma initially having $\beta = \beta_i$ and volume ΔV_i. Then if the pressure is increased to p the volume ΔV will be given by

$$\frac{p}{p_i} = \beta_i \left(\frac{\Delta V_i}{\Delta V}\right)^{5/3} + (1 - \beta_i)\left(\frac{\Delta V_i}{\Delta V}\right)^2 , \tag{3}$$

the terms on the right-hand side representing the partial pressures of the plasma and field respectively. This may be written in the form:

$$\frac{\Delta V}{\Delta V_i} = \left(\frac{p}{p_i}\right)^{-3/5} \left(\beta_i + (1 - \beta_i)\left(\frac{\Delta V}{\Delta V_i}\right)^{-1/3}\right)^{3/5} \tag{4}$$

which allows $\frac{\Delta V}{\Delta V_i} = f\left(\frac{p}{p_i}, \beta_i\right)$ to be obtained by successive approximations.

The volume of the whole system is then found by integration:

$$V = \int f\left(\frac{p}{p_i}, \beta_i\right) dV_i \tag{5}$$

To determine V as a function of p requires that the initial distribution of β is known, and this in turn depends on the method of creating the initial plasma. For LINUS systems, the rotating relativistic electron beam seems a promising method of inducing the plasma currents necessary to create the reversed-field configuration,[3] but at present little is known about the pressure distribution so formed. In this report we shall therefore consider two simple representative distributions, corresponding respectively to a uniform distribution of current density and a parabolic distribution centered on the neutral surface, which is taken to be at $r = R/2$. The two distributions are then:

$$\text{(a)}\quad j_\theta = \text{constant};\ \beta_i = 1 - 4x^2/R^2$$

$$\text{(b)}\quad j_\theta = 1 - 4x^2/R^2;\ \beta_i = 1 - (3x/R - 4x^3/R^3)^2 \tag{6}$$

where x is the distance from the neutral surface.

We have used equations (4) and (5) to derive V/V_i as a function of p/p_i for these two distributions. One significant result is that γ, the ratio of the specific heats of the plasma-field mixture, varies with the degree of compression. Here

$$\gamma = \frac{d(\ell n p)}{d(\ell n V)} \tag{7}$$

which is plotted against V_i/V, the volume compression ratio, in Fig. 2; this result will be used later in determining the liner trajectories.

III. Thermonuclear Reactions

We wish to compare the total fusion reaction rate W integrated over the plasma cross-section, with the corresponding rate in a $\beta = 1$ plasma. The comparison is greatly simplified by the fact that, for the D-T reaction, $\overline{\sigma v} \propto T^2$, with an accuracy of $\pm$ 10%, for 8 keV $< T <$ 30 keV. Provided that the final compressed plasma temperature is in this range, the local reaction rate is proportional to $n^2T^2 = p^2$ and we need only consider the radial distribution of <u>pressure</u>, without requiring that the density and temperature be specified independently. Thus,

$$\frac{W}{W_{\beta=1}} = \frac{\int \beta^2 \, dV}{\int dV} = \langle \beta^2 \rangle \tag{8}$$

Using the method of Section II, we have calculated $\langle \beta^2 \rangle$ as a function of V_i/V for the two initial distributions used previously. The results are given in Fig. 3. It is clear that there is a very significant reduction in the fusion rate at larger compressions, especially for distribution (b).

In Ref. 1 the liner trajectories were calculated assuming that the liner was compressing a $\beta = 1$ plasma with $\gamma = 5/3$. We now generalize this calculation to arbitrary γ, but retain the assumption of $\beta = 1$. The normalized equation of motion of the liner is now

$$\frac{da}{d\tau} = \frac{b}{a} \cdot \left(\frac{1 - a^{-2\gamma-2}}{\ell n(1+b^2/a^2)} \right)^{\frac{1}{2}} \tag{9}$$

where $a = r/r_o$, $\tau = tv_\infty/r_o$ and $v_\infty = (2p_o/(\gamma-1)b^2\rho)^{\frac{1}{2}}$. The subscripts o refer to values at peak compression. The energy per unit length is given by

$$E_o = \pi r_o^2 p_o/(\gamma - 1) \tag{10}$$

and the ratio Q of reaction energy released in one cycle to E_o is given by

$$\frac{Q}{B_o r_o \rho^{\frac{1}{2}}} = P(b, T_o, \gamma) = \frac{E_N(\gamma-1)^{3/2}}{32\pi^{\frac{1}{2}}} \frac{b}{T_o^2} \int \frac{\overline{\sigma v}}{a^2} d\tau \tag{11}$$

where E_N is the fusion energy per reaction, which we take as 17.6 MeV. As can be seen from equation (9), the integral is a function of γ. Since it has been shown previously[1] that P is rather insensitive to T_o (a consequence of the variation of $\overline{\sigma v}$ with T mentioned earlier) we take T_o = 20 keV and derive $P(\gamma)$ for b = 5, 8 and 11. The results are shown in Table 1. We find that the ratio $s_1 = P(\gamma)/P(5/3)$ is insensitive to b, and so it is plotted as a function of γ only in Fig. 4.

We may now combine the results of Figs. 2, 3 and 4 to obtain

$$s = \frac{P}{P_{\beta=1}} = s_1 \langle\beta^2\rangle \tag{12}$$

that is, the ratio of P for the reversed field pinch to its corresponding value for a $\beta = 1$ system. s is plotted as a function of V_i/V in Fig. 5.

IV. Discussion

The foregoing analysis has been arranged in such a way as to identify the separate contributions of the various effects listed in

Section I. The dominant effect is clearly the $\langle\beta^2\rangle$ term (Fig. 3). The effect of increased γ appears at first in Equation (11) as a $(\gamma - 1)^{3/2}$ term which, in the most favorable case ($\gamma = 1.95$, Fig. 2) leads to an increase in P of 1.7, but when the integral is included this is reduced to 1.22 (Fig. 4). The outcome of this exercise, culminating in Fig. 5, is to allow us to draw three general conclusions:

1) Uniform initial current distributions are to be preferred to peaked distributions.

2) Volumetric compression ratios should probably not exceed about 100-200.

3) For scaling studies of reversed-field LINUS systems, it is appropriate to take $s = 0.2$ as the value that could probably be achieved in practice. For the most optimistic projections, one might take $s = 0.25$.

The most important consequence of $\beta < 1$ is an increase in the radial dimensions of a LINUS reactor. For fixed Q, the increase is by a factor s^{-1}; however, as shown in Ref. 2, as the radius is increased, the Q necessary for a self-sustaining reactor cycle decreases, and the net increase in size is only by a factor $s^{-2/3}$, typically about 3.

V. References

1. A. E. Robson. Fundamental Requirements of a Fusion Feasibility Experiment Based on Flux Compression by a Collapsing Liner. NRL Memo Report 2616, July 1973.

2. A. E. Robson. A Simple Model of a LINUS Fusion System with a

Thick, Compressible, Resistive Liner. NRL Memo Report 3472, February 1977.

3. D. L. Bock, *et al.* Stabilized Imploding Liner Fusion Systems. 6th International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, FRG, 6-13 October 1976. Paper E-19-1.

Table 1

Table of $P(b, \gamma)$

γ \ $b =$	5	8	11
1.67	0.179	0.205	0.221
1.78	0.195	0.223	0.241
1.87	0.209	0.238	0.256
1.92	0.215	0.245	0.264
2.00	0.225	0.256	0.276

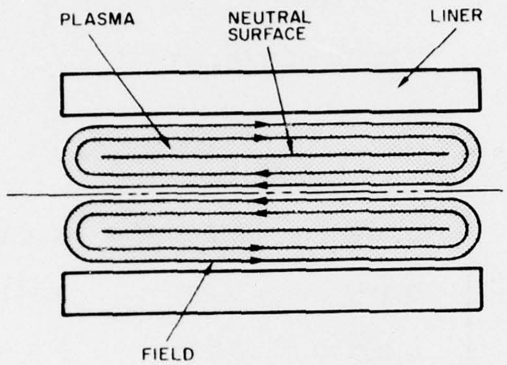

Fig. 1 — Reversed-field theta-pinch configuration

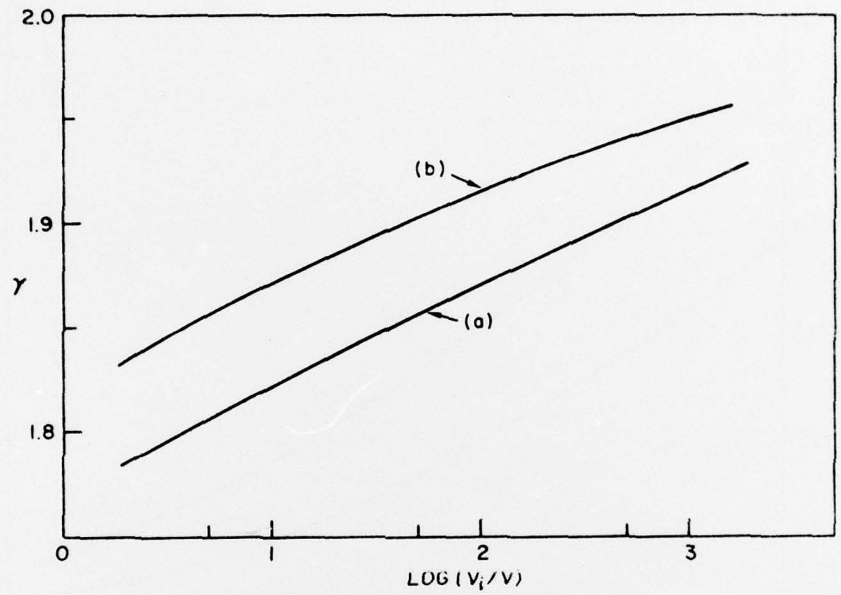

Fig. 2 — γ vs. Log (V_i/V)

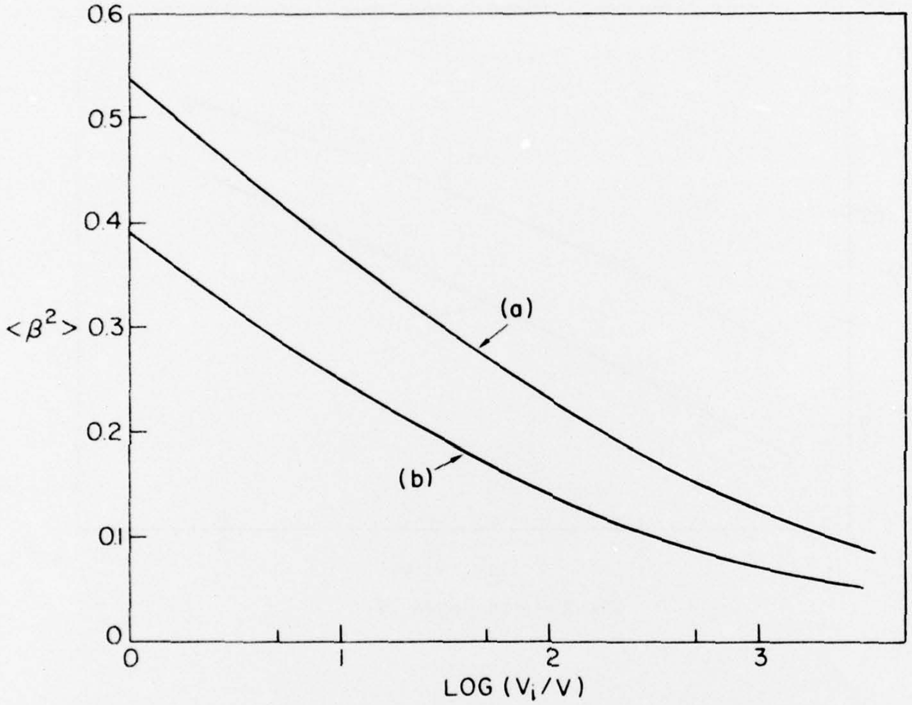

Fig. 3 — $\langle\beta^2\rangle$ vs. Log (V_i/V)

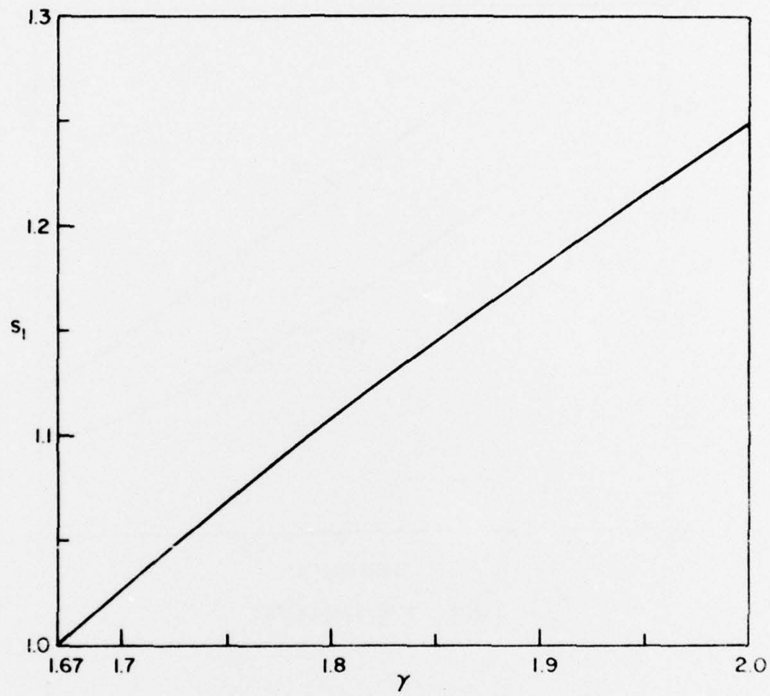

Fig. 4 — s_1 vs. γ

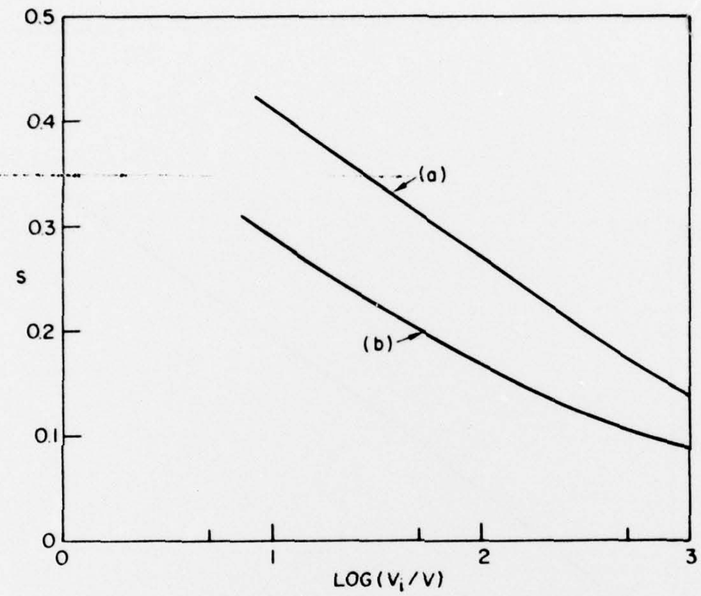

Fig. 5 — s vs. Log (V_i/V)